Sigma Notation:

Given the partial sum: $S_n = 5 + 10 + 15 + 20 + 25 + 30 + 35$ is easy enough to write.

But there is another way called **sigma notation**.

Here is an example of “this other way”:

The last value to plug into your formula

$$S_n = \sum_{n=1}^{8} 3 + 5(n-1)$$

Use this explicit formula to calculate terms

The first value to plug into your formula (then keep increasing by 1)

This says that

$$S_n = \{3 + 5(1 - 1)\} + \{3 + 5(2 - 1)\} + \{3 + 5(3 - 1)\} + \cdots + \{3 + 5(7 - 1)\} + \{3 + 5(8 - 1)\}$$

So $S_n = 3 + 8 + 13 + 18 + 23 + 28 + 33 + 38 = 164$

Another example:

$$S_n = \sum_{n=3}^{10} 4 \cdot (-2)^{n-1} = 16 - 32 + 64 - 128 + 256 - 512 + 1024 - 2048 = -1360$$

$$4 \cdot (-2)^{3-1} = 16 \qquad 4 \cdot (-2)^{10-1} = -2048$$

Let’s go back to our original problem $S_n = 5 + 10 + 15 + 20 + 25 + 30 + 35$.

Write this in sigma notation.

5, 10 15, 20, 25, 30, 35,... is an arithmetic sequence with the explicit formula: $a_n = 5 + 5(n - 1)$

For the first term, 5, n = 1. For the last term, 35, n = 7, so

$$S_n = \sum_{n=1}^{7} \llbracket 5 + 5(n-1 \rrbracket)$$

Example 2: Write $S_n = 3 + 9 + 27 + 81 + \cdots + 59049$ in sigma notation.

Use the explicit, geometric formula: $a_n = 3 \cdot 3^{n-1}$. $a_1 = 3$.

$59049 = 3 \cdot 3^{n-1}$ ❯ $19683 = 3^{n-1}$ ❯ $n - 1 = \log_3 19683$ ❯ $n - 1 = 9$ ❯ $n = 10$

$S_n = \sum_1^{10} 3 \cdot 3^{n-1}$

Example 3: Write $S_n = 1 + 2 + 3 + 4 + 5 + \cdots$ in sigma notation. (this is an infinite series)

$$S_n = \sum_{n=1}^{\infty} 1 + 1(n-1)$$